To Segregate or to Integrate: Education Politics and Democracy

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Abstract

In most democracies, the majority of education expenditures is financed by the government. In non-democracies, we observe a wide variation in the mix of public and private funding of education. In addition, countries with high inequality tend to rely more heavily on private schooling. We develop a theory which integrates private decisions on education and fertility with voting on public schooling expenditures. The theory is able to account for the facts mentioned above. Countries with high inequality exhibit more private education expenditures since rich people opt out of the public system. In non-democracies, concentration of political power leads to multiple equilibria in the determination of public education spending.

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1 Introduction

Public schooling is one of the most widespread social policies around the world today. Starting with industrializing European nations in the nineteenth century, nearly all countries have introduced compulsory schooling laws and public funding of education. However, despite the near universal involvement of the government, private and public funding of education continue to coexist. The share of private education funding varies greatly across countries, from only 1.9 percent of total spending in Norway, to 44.5 percent in Chile (1998, see Appendix A.1). Institutional arrangements concerning the funding of private schools also differ across countries. Private schools can be partly supported by the government as in France or New Zealand, or even entirely publicly funded as in Belgium, or rely exclusively on private funding as in the United States (Toma 1996).

The coexistence of public and private providers in some countries raises the question why societies make different choices about the mix of public and private funding of education. To address this question, we develop a heterogeneous agents model in which education is a private good, but people can vote to provide free public education which is financed through income taxes. Adults decide whether their children participate in public schooling. Parents who provide private education choose how much education to provide; in contrast, all children in the public system receive the same quality of education.

The decision of a parent between public and private education depends on the quality of tax-financed public schooling. In the model, the private option is chosen by people whose preferred quality of education far exceeds what is provided for free by the government. Public education spending, in turn, is determined through a vote of the adult population. The specific political mechanism that we employ is probabilistic voting. Relative to majority voting, the probabilistic model has the advantage that outcomes depend smoothly on every voter's preferences. In other words, results do not depend solely on the preferences of the median voter.

Since both the level of public funding and the decision whether to provide schooling privately are endogenous, we can analyze how the education-finance mix and the quality of education depend on the characteristics of an economy. In particular, we focus on two main determinants of political outcomes in a country, the distribution of income and the distribution of political power.

We find that when political power is evenly distributed, such as in a representative democracy, a unique political equilibrium exists. In this equilibrium it is never optimal to provide zero public schooling. The extent of public versus private funding depends on the income distribution. In a society with little inequality, all parents use public schooling. For increasing levels of inequality, first rich and ultimately some poor people choose private education for their children.[1] This result provides a contrast to the literature where a greater degree of inequality motivates more redistribution through the political process (see Alesina and Rodrik 1994, Persson and Tabellini 1994, and in particular Gradstein and Justman 1997 in an application to education). In our model the increased need for redistribution is offset by richer people who opt out of the public system and vote for less public schooling.

If the political system is biased towards low-income households, the results are qualitatively the same as in the case of an even distribution of political power. Since the public education system is redistributive, an electorate dominated by poorer people chooses higher levels of public education, but the link between inequality and the extent of public schooling remains intact.

If political power is concentrated among the rich, as in many dictatorships, there is no longer a guarantee that the voting outcome is unique. It is possible that a given population chooses either a high quality of public education with everyone participating in the public system, or a low quality of public education with the rich sending their children to private schools. The source of multiplicity is a complementarity between the rich voters' choice of whether to participate in public education, and their preferred quality of public schools.

An important feature of our model is that we allow for endogenous fertility. It has long been known that in the data fertility and education decisions are interdependent. By explicitly incorporating fertility choices, the model is able to generate realistic predictions for the relationship between the education regime and fertility differentials within the population. Endogenous fertility also turns out to be important from a theoretical perspective. In particular, a family's total expenditures on children are independent of the schooling choice, because fertility is lower in households opting

[1] This echoes the result of Besley and Coate (1991). Assuming that quality is a normal good, households who opt out of the public sector are those with higher incomes.

for private schools. This feature has consequences for the relation between the extent and the quality of public schooling, and helps to guarantee uniqueness of political outcomes in democracies.

The main predictions of our theory are consistent with evidence on the level of public and private schooling across countries. As described in Appendix A.1, inequality in the 1970s is a strong predictor of the share of private education funding in the 1990s. The correlation between the Gini coefficient in 1970 and the share of private education funding at the primary and secondary levels in 1998 is 0.55. We also find that fertility differentials between high and low educated mothers are higher in countries that rely more on private funds. Appendix A.2 documents that the relationship between inequality and private funding is also present across U.S. States.

Concerning the role of political power, we turn to the relationship between democracy and education funding. If we interpret democracies as countries with an even distribution of political power, while non-democracies are biased to the rich, our model implies that there is more scope for variation in education systems in non-democracies than in democracies. Indeed, using a cross section of 158 countries, we find that the variance of public spending across countries is smaller for democracies than for non-democracies (see Appendix A.3).

Consider, for example, the contrast of Saudi Arabia and the United Arab Emirates. Both are oil-rich countries which are similar in may respects, including low scores on the democracy index. One might expect that their education systems would be similar as well. In reality, however, Saudi Arabia spends 6.15 percent of GDP on public education, while the Emirates only spend 1.87 percent (1990-95 average). Our interpretation is that, in principle, a high-quality public schooling system could be supported in the Emirates as well. Given the low political power of the poor however, the quality of public education is so low that rich people prefer private schooling for their children. The rich therefore do not take an interest in the quality of public schooling, which perpetuates the existing regime of low public spending.

Our paper relates to an existing literature on the choice of public versus private schooling which relies on majority voting as the political mechanism (see Stiglitz 1974, Glomm and Ravikumar 1998, Epple and Romano 1996b, and Glomm and Patterson 2002). A recurring theme in this literature is the argument that if private alternatives to public schools exist, voters' preferences may fail to be single-peaked. In particular, very poor households, who are less willing than middle-income households to forgo consumption in lieu of better public schooling, will form a coalition with very rich households, who choose private schooling. A majority-voting equilibrium may still exist in this case, but expenditures are lower than what is preferred by the voter with median income. In our probabilistic voting model, equilibria are always guaranteed to exist. Moreover, the probabilistic voting setup is not restricted to democracies, since we can analyze what happens if all voting power is concentrated on the rich.

Our model makes predictions for the link between inequality in a country and the resulting education system and quality of education. A similar objective is followed by Fernández and Rogerson (1995), who consider a model where education is discrete (zero or one), partially subsidized by the government, and voters decide on the extent of the subsidy. Fernandez and Rogerson emphasize that in unequal societies, the very poor may forgo education alltogether. Since all voters are taxed, in this case public education constitutes a transfer of resources from the very poor and the very rich to the middle class, echoing the findings of Epple and Romano (1996a).

A different branch of the literature takes the schooling regime (public or private) as given, and analyzes the economic implications of each regime. Glomm and Ravikumar (1992) contrast the effects of public and private schooling systems on growth and inequality. In a country with little inequality, a fiscal externality created by public schooling leads to lower growth under public schooling than under private schooling. In unequal societies, however, public schooling can dominate, since more resources are directed to poor individuals with a high return on education. Similar conclusions are derived by de la Croix and Doepke (2004) in a framework which emphasizes the interdependence of fertility and education decisions of parents. The model of Glomm and Ravikumar (1992) has been extended by Bénabou (1996) to allow for local interactions between agents, such as neighborhood effects and knowledge spillovers.

In the next section, we introduce our model and analyze the political equilibrium. Section 3 describes how in a democratic country the choice of a schooling regime and the quality of schooling depend on the income distribution. Section 4 generalizes the voting process to allow for unequal political power. We show that multiple equilibria can arise in societies dominated by the rich. In Section 5, we extend the model to a dynamic framework and analyze the feedback from education choices to income and population dynamics. Section 6 concludes. The appendix summarizes empirical evidence and contains all proofs.

2 The Model Economy

Preferences and technology

The model economy is populated by people who care about consumption c, their number of children n, and their children's education h. We start with a static economy; in Section 5, we extend the analysis to an overlapping generations economy where today's children are tomorrow's adults. There are two types of people, skilled and unskilled. While our results generalize to an arbitrary number (or even a continuum) of skill levels, concentrating on two types will simplify the exposition. The two types are indexed by i and differ only in their wage w^i. The utility function is logarithmic:[2]

$$\ln(c^i) + \gamma \left[\ln(n^i) + \eta \ln(h^i)\right]. \tag{1}$$

Notice that parents care both about child quantity n^i and quality h^i. The parameter $\gamma \in \mathbb{R}_+$ is the overall weight attached to children. The parameter $\eta \in (0,1)$ is the relative weight of quality.[3] As we will see below, the tradeoff between quantity and quality is affected by the human capital endowment of the parent and by the schooling regime.

To attain human capital, children have to be educated by teachers. The wage of teachers equals the average wage in the population $\bar{w}$. Parents can choose from two different modes of education. First, there is a public schooling system, which provides a uniform education s to every student. Education in the public system is financed through an income tax v; apart from the tax, there are no direct costs to the parents.

[2] Any utility function representing homothetic preferences defined over the bundle (c^i, n^i, h^i) would lead to the same results.

[3] The parameter η cannot exceed 1 because the parents' optimization problem would not have a solution. More specifically, utility goes to infinity as parents choose arbitrarily high levels of education and arbitrarily low levels of fertility. In Section 5 we provide a different interpretation of the model, where the condition $\eta < 1$ implies decreasing returns to education. A similar condition can be found in Moav (2001).

The schooling quality s and the tax rate v are determined through voting, to be described in more detail later. Parents also have the possibility of opting out of the public system. In this case, parents can freely choose the education quality e^i, but they have to pay the teacher out of their own income. Since education e^i is measured in units of time of the average teacher, the total cost of educating n^i children privately is given by $n^i e^i \bar{w}$. We assume that education spending is tax deductible. While tax deductibility of education expenditures varies across countries, full deductibility simplifies the analysis because it implies that taxation does not distort the choice between quantity and quality of children. Apart from the education expenditure, raising one child also takes fraction $\phi \in (0, 1)$ of an adult's time. The budget constraint for an adult with wage w^i is then given by:

$$c^i = (1 - v) \left[w^i (1 - \phi n^i) - n^i e^i \bar{w} \right]. \tag{2}$$

Education is thus either private, e^i, or public, s. Effective education can be expressed as the maximum of the two: $h^i = \max\{e^i, s\}$. Of course, parents who prefer public education will choose $e^i = 0$.

The consumption good is produced by competitive firms using labor as the only input. We assume that the aggregate production function is linear in both labor inputs. The production setup does not play an important role in our analysis; the advantage of the linear production function is that wages are constant. Using A for unskilled and B for skilled, we have:

$$Y_t = w^A L^A + w^B L^B.$$

Here w^A and $w^B > w^A$ are the marginal product of each type. We can normalize $w^A = 1$ without loss of generality. The total input of the groups are given by L^A and L^B. The input of workers of type i is smaller than the total population P^i, since some adults work as teachers.

It is convenient to define the relative wage of a family of type i as $x^i = w^i / \bar{w}$. Substituting the budget constraint (2) into the utility function (1) allows rewriting the utility of household i as:

$$u^i[v, n^i, e^i, s] = \ln(1 - v) + \ln(x^i (1 - \phi n^i) - n^i e^i) + \gamma \ln n^i + \gamma \eta \ln \max\{e^i, s\}.$$

Relative wages are related to the size of the two groups. Denoting the sizes of the groups relative to group A by:

$$\xi^A = 1, \;\; \xi^B = \frac{P^B}{P^A}, \tag{3}$$

the average wage is given by:

$$\bar{w} = \frac{\sum_{i=A,B} P^i w^i}{\sum_{i=A,B} P^i} = \frac{1+\xi^B w^B}{1+\xi^B}, \tag{4}$$

which allows us to compute:

$$x^B = \frac{w^B(1+\xi^B)}{1+\xi^B w^B} \quad \in [1, w^B]. \tag{5}$$

We see from this equation that the wage of skilled people relative to the average, x^B, varies from w^B to 1 when ξ^B varies from 0 to infinity. Equation (4) also implies the following relation between x^B, x^A and ξ^A:

$$x^A = 1 + \xi^B(1 - x^B) \quad \in [1/w^B, 1]. \tag{6}$$

Timing of events and private choices

The level of public funding for education (s and v) is chosen by probabilistic voting among the adult population. The voters' preferences depend on their optimal fertility and education choices (n, e and ψ), which are made before voting takes place. In making these choices, agents have perfect foresight regarding the outcome of the voting process. This timing is motivated by the observation that public education spending can be adjusted frequently, while fertility cannot, and the choice between public versus private education entails switching costs that are non negligible.

At given expected policy variables v and s, the utility function u^i is concave in n^i. Within each group, some agents may choose public schooling, in which case their fertility rate is denoted $\hat{n}^i$, while others opt for private education; fertility for those in private schools is denoted as $\tilde{n}^i$. Both the skilled and unskilled parents planning to send their children to the public school choose the same fertility level:

$$\hat{n}^i = \arg\max_{n^i} u^i[v, n^i, 0, s] = \frac{\gamma}{\phi(1+\gamma)} \equiv \hat{n}. \tag{7}$$

The households planning to provide private schooling chose:

$$\begin{aligned} \tilde{n}^i &= \arg\max_{n^i} u^i[v, n^i, e^i, s] = \frac{x^i \gamma}{(1+\gamma)(e^i + \phi x^i)}, \\ e^i &= \arg\max_{e^i} u^i[v, n^i, e^i, s] = \frac{\eta \phi x^i}{1-\eta}. \end{aligned} \tag{8}$$

Private spending on education depends positively on relative wage x^i. Notice that e^i is independent from the outcome of the voting process, implying that the timing of choosing e^i does not matter. Replacing the optimal value for e^i in the fertility equation we find:

$$\tilde{n}^i = \frac{\gamma(1-\eta)}{\phi(1+\gamma)} \equiv \tilde{n} < \hat{n} \tag{9}$$

From equations (7) and (9) we see that parents choosing private education have a lower fertility rate. This result has a testable implication. In countries where public education is dominant, fertility differentials between high- and low-skilled mothers should be smaller than in countries with a large share of private education, which is what we find in Appendix A.1.

Lemma 1 (Constant parental spending on children)
For given s, v and w^i, parental spending on children (and therefore taxable income) does not depend on the choice of private versus public schooling and is equal to $\frac{\gamma}{1+\gamma} w^i$.

Lemma 1 implies that the tax base does not depend on the fraction of people participating in public schooling. This property will be important for generating uniqueness of equilibrium. The lemma relies on three assumptions: homothetic preferences, tax deductible education spending, and endogenous fertility. With endogenous fertility, parents choosing private schools have fewer children, keeping their total budget allocation to children in line with those choosing public schools.[4] This is a typical feature of endogenous fertility models.

[4]With fixed fertility, the resources allocated to children would be $w^i \phi n$ with public education and $w^i \phi n/(1-\eta)$ with private education.

The political mechanism

We denote the endogenous percentage of children of each group participating in the public education system as ψ^A and ψ^B. The public education system operates under a balanced-budget rule:

$$s \sum_{i=A,B} \xi^i \psi^i \hat{n}^i \bar{w} = v \sum_{i=A,B} \xi^i \left(x^i (1 - \phi \psi^i \hat{n}^i - \phi(1-\psi^i)\tilde{n}^i) - (1-\psi^i)\tilde{n}^i e^i \right) \bar{w}, \qquad (10)$$

with total spending on public education on the left-hand side and total revenues on the right-hand side. Since the level of schooling and taxes are linked through the budget constraint, the policy choice is one-dimensional.

The level of public expenditures, and hence taxes, is chosen through probabilistic voting. Assume that there are two political parties, p and q. Each one proposes a policy: s^p, s^q. The gain (or loss) of voter i if party q wins the election instead of p is $u^i[v^q, n^i, e^i, s^q] - u^i[v^p, n^i, e^i, s^p]$. Instead of assuming that voter i votes for q every time this difference is positive as in the median voter model, probabilistic voting theory supposes that this vote is uncertain. More precisely, the probability that voter i votes for party q is given by

$$F^i \left(u^i[v^q, n^i, e^i, s^q] - u^i[v^p, n^i, e^i, s^p] \right),$$

where F^i is a differentiable cumulative distribution function. This function captures the idea that voters care about an "ideology" variable in addition to the specific policy measure at hand, i.e., the quality of public schooling. The presence of a concern for ideology, which is independent of the policy measure, makes the political choice less predictable (see Persson and Tabellini (2000) for different formalizations of this approach).

Since the share of each type voting for a given party varies continuously with the proposed policy platform, probabilistic voting leads to smooth aggregation of all voters' preferences, instead of depending solely on the preferences of the median voter. Party q maximizes its expected vote share, which is given by $\sum_i \xi^i F^i(\cdot)$. Party p acts symmetrically, and, in equilibrium, we have $s = s^q = s^p$. The maximization program of each party implements the maximum of the following weighted social welfare function:[5]

$$\sum_{i=A,B} \xi^i \, (F^i)'(0) \, u^i[v, n^i, e^i, s] = 0.$$

The weights $(F^i)'(0)$ capture the responsiveness of voters to the change in utility. In particular, a group that has little ideological bias cares relatively more about economic policy. Such groups are therefore targeted by politicians and enjoy high political power. In addition, political power may also depend on other features of the political system such as voting rights. We will capture the political power of each group by a single parameter θ^i. This includes the extreme cases of representative democracy with equal responsiveness ($\theta^A = \theta^B$), and dictatorship of the rich ($\theta^A = 0$). Accordingly, the objective function maximized by the probabilistic voting mechanism is given by:

$$\Omega[s] \equiv \sum_{i=A,B} \xi^i \theta^i \left(\psi^i u^i[v, \hat{n}, 0, s] + (1 - \psi^i) u^i[v, \tilde{n}, e^i, 0] \right). \tag{11}$$

The maximization is subject to the government budget constraint (10).

We start by assuming that all individuals have the same political power, i.e. $\theta^A = \theta^B = 1$, implying that the weight of each group in the objective function is simply given by its population share. The role of this assumption will be further investigated in Section 4. It can be checked that $\Omega[s]$ is strictly concave. Taking the first-order condition for a maximum and solving for s yields:

$$s = \frac{\eta\phi(1 + \xi^B)}{1 + \xi^B + \gamma\eta(\psi^A + \xi^B \psi^B)} \equiv s[\psi^A, \psi^B]. \tag{12}$$

From this expression we can see that s is decreasing in both participation rates ψ^A and ψ^B. This reflects a congestion effect: when more children participate in public schools, spending per child is reduced. Looking at the corresponding tax rate,

$$v = \frac{\eta\gamma(\psi^A + \xi^B \psi^B)}{1 + \xi^B + \gamma\eta(\psi^A + \xi^B \psi^B)}, \tag{13}$$

we observe that a rise in participation is followed by a less than proportional rise in taxation. Since, by Lemma 1, the taxable income is unaffected by increased participa-

[5] This result was first derived by Coughlin and Nitzan (1981). The same framework can also be derived within the setup of lobbying models, see Bernheim and Whinston (1986).

tion, this translates into lower spending per child.

The equilibrium

So far, we have taken the fractions ψ^A and ψ^B of agents participating in public education as given, and solved for the corresponding voting outcome concerning the quality of public schools. In equilibrium, the choice whether or not to participate in public schooling has to be optimal. This leads to the following definition of an equilibrium:

Definition 1 (Equilibrium)
An equilibrium is a vector of individual variables $(\hat{n}^i, \tilde{n}^i, e^i, \psi^i)_{i=A,B}$ *and aggregate variables* (s, v) *such that equations (7)-(13) hold, and the following conditions are satisfied:*

$$\forall i,\ if \begin{cases} \psi^i = 1 & : \quad u^i[v, \hat{n}, 0, s] \geq u^i[v, \tilde{n}, e^i, 0] \\ 1 > \psi^i > 0 & : \quad u^i[v, \hat{n}, 0, s] = u^i[v, \tilde{n}, e^i, 0] \\ \psi^i = 0 & : \quad u^i[v, \hat{n}, 0, s] \leq u^i[v, \tilde{n}, e^i, 0]. \end{cases} \tag{14}$$

The first constraint says that if public education yields higher utility for group i, everyone in group i uses the public education system. The second constraint says that for group i to split between the two types of schools they need to be indifferent between them. The third constraint applies when group i prefers private education.

A first result is that parents with high human capital are more demanding in terms of public education quality, or, in other words, quality is a normal good:

Lemma 2 (Opting out decision)
There exist s^A *and* s^B*, satisfying* $0 < s^A < s^B$*, such that type i opts out of public education if and only if* $s < s^i$*.*

An implication of the above lemma is that if some skilled people choose the public regime, all unskilled love it. Similarly, if at least some unskilled people choose private education, all skilled household choose it as well.

Lemma 3 (Coverage of public education)
Denote the coverage of public education as $\Psi = \psi^A + \psi^B \in [0,2]$. ψ^A and ψ^B satisfy the following relationships

$$\begin{aligned}(\psi^B > 0) &\Rightarrow (\psi^A = 1)\\ (\psi^A < 1) &\Rightarrow (\psi^B = 0),\end{aligned}$$

and are therefore uniquely determined by Ψ.

An implication of the Lemma is that potential equilibria can be indexed by Ψ. We can now define the function:

$$\Delta^i[\Psi] = u^i[v, \hat{n}, 0, s] - u^i[v, \tilde{n}, e^i, 0]$$

which expresses the difference of utilities between public and private education for group i as a function of Ψ, where s and v depend on Ψ. $\Delta^i[\Psi]$ is not well defined at $\Psi = 0$, since here measure zero of agents use public schooling, so that positive public schooling can be provided at zero taxes. We therefore extend the function as follows:

$$\Delta^i[0] = \lim_{\Psi \to 0} \Delta^i[\Psi].$$

Extended this way, the function $\Delta^i[\Psi]$ is continuous. We can now define a mapping F from (Δ^A, Δ^B) to the interval $[0,2]$, which gives the set of values for Ψ consistent with Δ^A and Δ^B in equilibrium:

$$F(\Delta^A, \Delta^B) = \begin{cases} 2 & \text{for} \quad \Delta^A > 0, \Delta^B > 0 \\ [1,2] & \text{for} \quad \Delta^A > 0, \Delta^B = 0 \\ 1 & \text{for} \quad \Delta^A > 0, \Delta^B < 0 \\ [0,1] & \text{for} \quad \Delta^A = 0, \Delta^B < 0 \\ 0 & \text{for} \quad \Delta^A < 0, \Delta^B < 0. \end{cases}$$

The combined mapping $F(\Delta^A, \Delta^B)$ maps the interval $[0,2]$ into itself, and equilibria are given by fixed points of this mapping. We can now examine existence and uniqueness of equilibria.

Proposition 1 (Existence of equilibria)
An equilibrium exists.

The following corollary shows that the private regime is never an equilibrium.

Corollary 1 (Equilibrium coverage of public education)
In equilibrium we always have $\Psi > 0$.

Both differences $u^i[v, \hat{n}, 0, s] - u^i[v, \tilde{n}, e^i, 0]$ are monotonically decreasing in ψ^A and ψ^B, because v drops out and s depends negatively on ψ^A and ψ^B through the congestion effect. Consequently, the functions Δ^A and Δ^B are monotonically decreasing in Ψ. This will ensure uniqueness of equilibrium.

Proposition 2 (Uniqueness of the equilibrium)
The equilibrium is unique.

The uniqueness result relies on the assumption that fertility is endogenous. If one assumes to the contrary that fertility is exogenous and constant, Lemma 1 no longer holds and the tax basis increases with the ψ^i. Computing the total effect of the ψ^i on s, s still decreases with ψ^A but now increases with ψ^B: for skilled parents the tax basis effect dominates the congestion effect. Consequently, uniqueness is no longer guaranteed. This result underlines the importance of accounting for the interaction of fertility and education decisions.

3 Comparing the Education Regimes

We showed in Corollary 1 that there are always some parents who choose public education for their children. Depending on the coverage of the public education system, this leaves four cases to be considered.

Regime	Ψ	ψ^A	ψ^B
Public	2	1	1
Partial segregation (1)	$\in (1,2)$	1	$\in (0,1)$
Segregation	1	1	0
Partial segregation (2)	$\in (0,1)$	$\in (0,1)$	0

In the public regime, all children go to public school. Under segregation, all skilled parents send their children to private school, while unskilled use public schools. In the two partial segregation regimes, either the skilled (1) or the unskilled (2) parents are indifferent between public and private schools. We now characterize these regimes in more detail, with a special focus on the quality of public education s. We also examine under which conditions each regime arises in equilibrium.

We have seen in equation (12) that quality of public education depends negatively on participation rates, due to a congestion effect. As a consequence, we have the following proposition.

Proposition 3 (Quality of public education)
Public education spending per child s is:

$$s = \frac{\eta\phi}{1+\gamma\eta} \quad \text{in the public regime,} \tag{15}$$

$$s = \frac{\eta\phi(1+\xi^B)}{1+\xi^B+\gamma\eta} \quad \text{in the segregated regime.} \tag{16}$$

Quality in public schools is higher in the segregation regime than in the public regime. In the segregation regime, the quality of education decreases with the mass of unskilled people in the economy.

In the public regime, the quality of education s depends neither on the relative incomes x^i nor on the weight of each group ξ^i. In the segregation regime, the quality of public schooling increases in the relative number ξ^B of skilled people in the population, since the tax base is wider if there are more skilled people. This can have dynamic consequences: if the number of unskilled people tends to increase over time in a given country, our model predicts that the quality of public education will fall.

The model also has interesting implications in the partial segregation regimes. In these cases, one group of people is indifferent between both types of schools. This implies that the quality they receive from public schools is lower than the one from private schools, since the gap between the two has to compensate for higher costs of private education. This result is consistent with the literature devoted to the estimation of the relative quality of private education, correcting for the effect of higher social class of the pupils in the private sector. The majority of the results suggest that controlling for sample selectivity reduces the achievement advantage of private school students over public school students, but does not eliminate it.[6]

We now turn to the question under which condition each education regime arises. The following proposition summarizes the results.

Proposition 4 (Occurrence of education regimes)
Whether public schooling can arise in equilibrium depends on the preference parameters γ and η. Let $\delta = (1-\eta)^{1-\frac{1}{\eta}}$ and $\hat{\gamma} = \delta/\eta$.
If $\gamma > \hat{\gamma}$, public education is not an equilibrium outcome. Segregation arises if the conditions

$$x^B > \frac{\delta(1+\xi^B)}{1+\xi^B+\gamma\eta} \tag{17}$$

and

$$x^B \geq \frac{1+\xi^B}{\xi^B}\left(1 - \frac{\delta}{1+\xi^B+\gamma\eta}\right) \tag{18}$$

are satisfied. Partial segregation (1) (skilled are indifferent) arises if (17) is violated, and partial segregation (2) (unskilled are indifferent) arises if (18) is violated.

If $\gamma < \hat{\gamma}$, partial segregation (2) (unskilled are indifferent) never arises. Public education is an equilibrium if

$$x^B \leq \frac{\delta}{1+\gamma\eta} \tag{19}$$

[6] See Kingdon (1996) for India, Bedi and Garg (2000) for Indonesia, Alderman, Orazem, and Paterno (2001) for Pakistan, Vandenberghe and Robin (2003) for Brazil and Neal (1997) for Catholic U.S. schools. Some other studies find no difference between private and public schools performances (see Goldhaber (1996)).

holds, segregation arises if (17) is satisfied, and partial segregation (1) arises if both (19) and (17) are violated.

Corollary 2 (Inequality and segregation)
High income inequality leads to segregation, i.e., there is always a threshold for x^B above which segregation is the only equilibrium.

Figure 1 depicts the conditions under which each education regime arises. Two cases can be distinguished depending on the value of γ. If $\gamma < \hat{\gamma}$, then a country with low inequality (low x^B) will choose a public education regime. A country with high inequality will opt for segregation. In between there is a zone in which there is a public sector in which all unskilled and some skilled participate. If $\gamma > \hat{\gamma}$, the public regime is never an equilibrium. Again, high inequality leads to segregation. Moreover, if the share of skilled households in the population is low (low ξ^B), unskilled people are indifferent between using public and private schools, so that the private sector serves children from both groups.

4 Political Power and Multiple Equilibria

In this section, we relax the earlier assumption that both groups carry equal weight in the voting process. We will see that if political power is concentrated among high-income individuals, multiple equilibria can arise. We can normalize $\theta^B = 1$ without loss of generality and vary θ^A to capture variations in the bias of the political system. Taking the first-order condition for a maximum of the objective function (11) and solving for s yields:

$$s = \frac{\eta\phi(1+\xi^B)(\theta^A\psi^A + \psi^B\xi^B)}{(\psi^A + \psi^B\xi^B)[\theta^A + \xi^B + \gamma\eta(\theta^A\psi^A + \xi^B\psi^B)]} \equiv s_\theta[\psi^A, \psi^B]. \tag{20}$$

The corresponding tax rate is:

$$v = \frac{\eta\gamma(\theta^A\psi^A + \xi^B\psi^B)}{\theta^A + \xi^B + \gamma\eta(\theta^A\psi^A + \xi^B\psi^B)}. \tag{21}$$

In the new formulation with variable voting power, Lemma 3 and Proposition 1 (existence of equilibrium) still go through. However, whether the equilibrium is unique now depends on the weight θ^A of group A in the political system. If the political bias favors the poor ($\theta^A \geq 1$), uniqueness is still guaranteed:

Proposition 5 (Uniqueness for $\theta^A \geq 1$)
If the parameter θ^A satisfies

$$\xi^B(\theta^A - 1) + \theta^A(\theta^A(1+\gamma\eta) - 1) > 0 \tag{22}$$

there exists a unique equilibrium. $\theta^A \geq 1$ is sufficient for (22) to hold.

For $\theta^A < 1$, multiple equilibria may arise. It is also no longer true that the private regime never exists (as we showed for $\theta^A = 1$ in Corollary 1). Indeed, as Ψ goes to zero, public spending becomes:

$$\lim_{\psi^A \to 0} s_\theta[\psi^A, 0] = \frac{\eta\phi(1+\xi^B)\theta^A}{\theta^A + \xi^B}.$$

The private regime exists if at this schooling level the unskilled prefer private over public schooling. The condition for the existence of the private regime is:

$$\Delta^A(0) = \gamma\left(-\eta\ln(x^A) - (1-\eta)\ln(1-\eta) + \eta[\ln(\theta^A(1+\xi^B)) - \ln(\theta^A + \xi^B)]\right) < 0.$$

This condition is satisfied if θ^A lies below a certain threshold:

$$\theta^A < \frac{x^A\xi^B}{(\delta - x^A) + \delta\xi^B}.$$

If the voting power of the unskilled is sufficiently low, the private regime exists. Intuitively, if the unskilled have little political influence, the quality of public schooling is very low. Therefore private schooling becomes attractive to both types of agents.

To show that multiple equilibria can arise, we concentrate on the zone where public schooling is the unique equilibrium when $\theta^A = 1$. We establish that in this zone there are at least three equilibria for a sufficiently strong bias to the rich (θ^A sufficiently small).

Proposition 6 (Multiplicity of equilibria for $\theta^A < 1$)
If θ^A, γ, and x^B satisfy the conditions

$$\theta^A < \frac{x^A \xi^B}{(\delta - x^A) + \delta \xi^B}, \qquad \gamma < \hat{\gamma}, \quad \textit{and} \quad x^B < \delta/(1+\gamma\eta),$$

there are at least three equilibria.

Figure 2 illustrates Proposition 6. The private and public regimes are equilibria. There is also a partial segregation regime with $\Psi \in (1,2)$.

The possibility of multiple equilibria exists because we assumed that people have to pre-commit to fertility and education choices before the vote takes place. If all decisions were taken simultaneously, the voting process would lead to the same outcome as the weighted social planning problem, which is unique. Pre-commitment generates multiplicity in this setting, but not in the version with equal political weights, because there is now a strategic complementarity between the education choices of skilled people through the quality of public schools.[7] When all skilled people use private schools, an individual skilled person does not want to switch to the public system since the quality of the public schooling is low. If, however, all skilled people were to switch together to the public system, they would vote for a much higher quality of public schools; in that case it would be rational to stay in the public system. Here, the political bias towards the rich offsets the congestion effect resulting from higher participation to public schooling. Provided that there is a strong concentration of political power, the model can account for the fact that countries with similar characteristics can choose different educational systems.

Our result can be compared to other voting models where multiple equilibria arise. In Saint-Paul and Verdier (1997) there is majority voting on a capital income tax. If political power is unequally distributed, and is biased in favor of households having better access to world capital markets, expectations-driven multiple equilibria are possible. In a dynamic majority voting framework, Hassler, Rodriguez Mora, Storesletten, and Zilibotti (2003) assume that young agents base their education decisions on expectations over future redistribution. Self-fulfilling expectations can lead to either high or

[7] When actions are strategic complements, the utility of those taking the action depends positively on how many people take the action. Classic examples are Matsuyama (1991) for increasing returns, Katz and Shapiro (1985) for network externalities, and Diamond and Dybvig (1983) for bank runs.

low redistribution equilibria. Finally, there are other political economy models that do not have indeterminacy of equilibrium but display multiple steady states (see for example Bénabou 2000). Initial conditions, as opposed to self-fulfilling expectations, determine which steady state the economy approaches.

The next proposition shows that despite the possibility of multiple equilibria, the coverage of public schooling is never higher in societies dominated by the rich than in democracies:

Proposition 7 (Coverage of public education as a function of θ^A)
Let $\hat{\Psi}$ be the equilibrium coverage of public education for $\hat{\theta}^A \geq 1$, and $\hat{v}$ the corresponding tax rate. If $\tilde{\Psi}$ and $\tilde{v}$ are an equilibrium coverage and a tax rate for a $\tilde{\theta}^A \leq \hat{\theta}^A$, then we have

$$\begin{aligned} \tilde{\Psi} &\leq \hat{\Psi}, \\ \tilde{v} &\leq \hat{v}. \end{aligned}$$

In summary, if the political system is tilted towards the poor, the equilibrium is unique and both the coverage of and spending on public education increase with the political power of the poor. In contrast, if the rich wield more power than the poor, multiple equilibria may arise. In any such equilibrium, the coverage of and spending on public education cannot be higher than in the outcome with equal political weights. If the influence of the poor is sufficiently low, entirely private education systems are possible.

5 The Dynamics of Education Regimes

In the previous sections we have analyzed how the distribution of income and political power shape education systems. We now consider how education feeds back into income and population dynamics. Two dynamic links are key for this relationship. First, a child's probability of becoming skilled depends on its education. Second, the weight of each group in the population is influenced by the fertility of both groups. We will concentrate on dynamics in a democracy (where outcomes are unique), and therefore assume $\theta^A = 1$.

To introduce probabilities of becoming skilled, we rewrite the utility function (1) as follows:

$$\ln(c_t^i) + \gamma \ln(n_t^i \pi_t^i). \tag{23}$$

Adults now care about the probability π_t^i that their children become skilled. This probability depends on the education h_t^i they receive. Specifically, given education h, the probability of becoming skilled is given by:

$$\pi_t^i = \tau^i \, h_t^\eta.$$

If we plug this function into (23) we recover the original form of the utility function (1) up to a constant. We denote the complement probability $1 - \pi_t^i$ as $\bar{\pi}_t^i$. Note that equation (8) can be used to define an upper bound on τ^i such that the probabilities belong to $(0, 1)$. Since x^i is bounded above by w^i, τ^i should belong to $(0, \left((1-\eta)/(\eta\phi w^i)\right)^\eta)$. The parameter η now measures the elasticity of success with respect to the educational investment. Writing probabilities as a function of education, the population evolves according to:

$$\begin{aligned}
P_{t+1}^A &= \left[\hat{n}\psi_t^A \bar{\pi}^A(s_t) + \tilde{n}(1-\psi_t^A)\bar{\pi}^A(e_t^A)\right] P_t^A \\
&+ \left[\hat{n}\psi_t^B \bar{\pi}^B(s_t) + \tilde{n}(1-\psi_t^B)\bar{\pi}^B(e_t^B)\right] P_t^B \qquad (24) \\
P_{t+1}^B &= \left[\hat{n}\psi_t^A \pi^A(s_t) + \tilde{n}(1-\psi_t^A)\pi^A(e_t^A)\right] P_t^A \\
&+ \left[\hat{n}\psi_t^B \pi^B(s_t) + \tilde{n}(1-\psi_t^B)\pi^B(e_t^B)\right] P_t^B \qquad (25)
\end{aligned}$$

In the previous sections we have seen that all decision problems in the model are static in nature. An intertemporal equilibrium is therefore a sequence of time-t equilibria held together by the laws of motion of the state variables. Existence and uniqueness of time-t equilibria imply the same properties for intertemporal equilibria.

Definition 2 (Intertemporal equilibrium)
Given initial conditions $(P_0^i)_{i=A,B}$, an intertemporal equilibrium is a sequence of time-t equilibria with $\{P_t^i\}_{t\geq 0,\, i=A,B}$ satisfying equations (24)-(25) at all dates $t > 0$.

Proposition 8 (Existence and uniqueness of intertemporal equilibria)
Given initial conditions $\{P_0^i\}_{i=A,B}$, an intertemporal equilibrium exists and is unique.

We now analyze the dynamic behavior of the economy. The evolution of the population is described by equations (24)-(25). Rewriting the law of motion in terms of ξ_t^B gives:

$$\xi_{t+1}^B = \frac{\hat{n}\psi_t^A \pi^A(s_t) + \tilde{n}(1-\psi_t^A)\pi^A(e_t^A) + \left[\hat{n}\psi_t^B \pi^B(s_t) + \tilde{n}(1-\psi_t^B)\pi^B(e_t^B)\right]\ \xi_t^B}{\hat{n}\psi_t^A \bar{\pi}^A(s_t) + \tilde{n}(1-\psi_t^A)\bar{\pi}^A(e_t^A) + \left[\hat{n}\psi_t^B \bar{\pi}^B(s_t) + \tilde{n}(1-\psi_t^B)\bar{\pi}^B(e_t^B)\right]\ \xi_t^B}$$
$$\equiv \Gamma(\xi_t^B) \quad (26)$$

Proposition 9 (Global dynamics)
The dynamics described by $\xi_{t+1}^B = \Gamma(\xi_t^B)$ are bounded, and always admit a steady state in $\mathbb{R}_+$.

The dynamics of ξ^B do not always converge to the steady state whose existence is guaranteed by the proposition. Since $\Gamma(0) > 0$ and $\Gamma(\xi) < \xi$ for large ξ, there is always a steady state $\bar{\xi}$ for which $\Gamma'(\bar{\xi}) < 1$. There are examples where the steady state is locally stable ($-1 < \Gamma'(\xi) < 1$), and other examples where $\Gamma'(\xi) < -1$ and the steady state is unstable. In the latter case, deterministic ever-lasting fluctuations may occur. Figure 3 provides such an example.[8] The lower panel depicts the law of motion; it has two increasing and one decreasing segments. The upper panel shows the education regime that each ξ_t^B corresponds to. The declining schedule in the upper panel depicts the mapping (5) from ξ_t^B into x_t^B. We see that the declining part of the law of motions corresponds to values for ξ_t^B which gives rise to the partial segregation regime. The increasing segment of the law of motion above the 45 degrees line corresponds to segregated education and the increasing segment below the 45 degrees line arises under public education. The steady state (thick dot) lies in the decreasing segment and is locally unstable. Instead there exists a limit cycle of period 2 (hollow dots) to which dynamics converge. The economy reverts back and forth between the public and the segregation regime, suggesting periodic swings in the balance between the public and private provision of education.

Further insights on dynamics can be obtained within the public regime. We now provide conditions under which a locally stable steady state exists in this regime. To specialize to public education, we replace ψ_t^A and ψ_t^B in equation (26) by 1, and s_t by

[8] The parameters are: $\eta = 0.4$, $\phi = 0.075$, $\gamma = 0.2$, $w^B = 3$, $\tau^B = 1.7$, and $\tau^A = 0.75$.

its optimal value $s[1,1]$ given in equation (15):

$$\xi^B_{t+1} = \frac{\pi^A(s[1,1]) + \pi^B(s[1,1])\xi^B_t}{\bar{\pi}^A(s[1,1]) + \bar{\pi}^B(s[1,1])\xi^B_t}. \tag{27}$$

Equation (27) can be solved for a unique positive steady state:

$$\xi^B = \frac{\pi^A(s[1,1])}{1 - \pi^B(s[1,1])}. \tag{28}$$

Using equation (5), the corresponding level of x^B is:

$$x^B = \frac{w^B\left(1 - (\tau^B - \tau^A)\,(s[1,1])^\eta\right)}{1 - (\tau^B - \tau^A w^B)\,(s[1,1])^\eta}. \tag{29}$$

We now provide a condition for a steady state inside the public regime and show that it is always locally stable.

Proposition 10 (Dynamics with public education)
If:

$$w^B < \frac{\delta\left(1 - \tau^B\,(s[1,1])^\eta\right)}{(1+\gamma\eta)\left(1 - \tau^B\,(s[1,1])^\eta\right) + \tau^A(1+\gamma\eta-\delta)\,(s[1,1])^\eta},$$

the dynamics of the public regime admit a steady state given by equations (28) and (29), which is locally stable.

If the skill premium w^B is too large, there is no steady state in the public regime. Otherwise, if an economy starts in the public regime not too far away from the steady state, it will converge to the steady state. Convergence is monotonous if $\tau^B > \tau^A$, i.e. if children of skilled parents have a higher probability of becoming skilled than children of unskilled parents for a given level of education. This stability result is consistent with the fact that countries having a public regime tend to stay in this regime (see Appendix A.1). In the remaining education regimes, analytic solutions for steady states and stability are not available. Private spending e^A_t and e^B_t and public spending s_t will all depend on ξ^B_t, which makes equation (26) depend on higher powers of ξ^B_t.

6 Conclusions

The education of its citizens is one of main areas of government intervention in every country in the world. At the same time, the government is generally not the only provider of education; education systems often display a juxtaposition of public and privately funded institutions. The degree of private involvement in the provision of education varies a great deal over different countries, going from fully public systems as in some European countries to segregated systems as in parts of the U.S. In this paper, we try to understand how countries choose the mix of public and private education.

We first conclude that high inequality maps into a segregated education system. In a segregated system, the quality of public schools is sufficiently low for rich households to prefer paying for private schools to enhance the education of their children. When inequality is low, on the other hand, the rich decide to send their children to public schools, so that they avoid paying for education twice (first through taxes, second through private schools). The prediction of a strong relationship between inequality and the extent of public schooling is in line with empirical evidence. In both cross-country data and cross-state data in the U.S., we find that public spending on education is negatively related to income inequality.

Turning to the role of political power, we find that the quality and extent of public schooling generally increases with the political weight of the poor. In addition, if the poor are given at least equal weight in the political system, there is a unique equilibrium outcome. In societies that are politically dominated by the rich, on the other hand, multiple equilibria may arise. The reason is that when the rich are in charge, there is a complementarity between the number of rich people participating in public schools and their quality. For given initial conditions, such a country may either have a high-quality public schooling system in which many or all of the rich participate, or a low-quality system with all the rich using private schools. Despite the multiplicity, however, we find that spending on public education is never higher in a society dominated by the rich than in an otherwise identical economy where the poor have equal power. The model therefore provides an explanation for the observation that non-democratic countries spend on average a smaller fraction of GDP on public education than democracies, whereas the variance of spending across countries is higher.

While we have concentrated on cross-section evidence, another important question for future research is why public education was first introduced in the nineteenth century during the second phase of the Industrial Revolution. Galor and Moav (2001) argue that in this period capitalists started to have an interest in public education, because of complementarities between physical and human capital. Therefore, technological change strengthening this complementarity may have contributed to the introduction of public schooling. Galor, Moav, and Vollrath (2003) extend this analysis by distinguishing different sources of wealth. If land is less complementary to human capital than physical capital, a conflict of interest arises between land-owners and capitalists. The outcome of this conflict depends on the distribution of wealth and land-ownership.

In our model, public schooling always arises if political power is equally shared. The theory therefore points to the expansion of voting rights in the nineteenth century as a key explanation for the introduction of public schooling. This still leaves open the question why voting rights were expanded in the first place. The theory of Galor and Moav (2001) offers one potential explanation. Acemoglu and Robinson (2000), however, point in a very different direction: the rich shared power in order to avoid the threat of a revolution. In either case, given that the poor did gain political influence, in our model the introduction of public schooling is a necessary consequence. Once public education is in place, the size of the public system depends on the evolution of the income distribution. To this end, the model points to the declining income inequality observed around the turn of the century as a potential explanation for the large expansion of public education that followed its initial introduction.

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A Data Appendix

A.1 OECD Data on Private Funding

The OECD provides data on the relative proportions of public and private investment in education, distinguishing two levels of education, primary/secondary and tertiary, and covering the period 1985-1998. Apart from 1998, only OECD countries are covered. The number of countries varies from six in 1985 to 31 in 1998. In most countries, private sector expenditure is comprised mainly of household expenditures on tuition and other fees. The exception is Germany, where nearly all private expenditure is accounted for by contributions from the business sector to the system of apprenticeship at the upper secondary level. For primary and lower secondary education, there is little private funding in Germany.

According to the OECD data, the scale of private-sector funding of education is increasing over time. This general trend, however, hides a variety of different patterns. Countries that have had predominantly public schooling for a long time tend to stay in this regime. Among the eight countries that had a share of public spending larger than 90 percent in the 1980s, seven are still in this situation 15 years later (Austria, Denmark, Netherlands, Portugal, Sweden, Finland, Italy). Only Canada left the group. Among the countries with a lower share of public funding, a majority experienced a decline in the share of public education (Australia, Canada, Hungary, Ireland, Japan, Korea, USA), while a smaller number expanded the public share (France, Germany, Spain, Iceland, Mexico).

In 1998, the data set contains information on a number of non-OECD countries (Norway, Israel, Uruguay, Czech Republic, Switzerland, Turkey, Argentina, Indonesia, Chile, Peru, Philippines, and Thailand). With observations on 31 countries, we can investigate whether inequality is a good predictor of private funding. Computing the correlation between Gini inequality coefficient estimated in 1970 by Deininger and Squire (1996) and private funding in 1998, we find that the correlation is positive and relatively strong (0.44 or 0.45 if Germany is excluded from the sample), even stronger when we consider only primary and secondary levels (0.55 or 0.59 if Germany is excluded). Figure 4 presents the cross plot of the private share in primary and secondary education with the Gini coefficient.

An implication of our theory is that countries that use private resources in education more intensively should be characterized by higher fertility differentials. As pointed out by de la Croix and Doepke (2003), such endogenous fertility differentials amplify the negative effect of inequality on growth. Table 1 provides average fertility differentials for countries with high and low use of private resources in primary and secondary education. Differential fertility is defined as the difference in fertility between women from the highest education group and women from the lowest group (those data from various surveys are summarized in Kremer and Chen 2002). The correlation coefficient between the two variables is 0.49, which is significantly different from zero at the 5 percent level with 15 observations.

	Share of Private Funding	Differential Fertility
High (> 15)	29.50	1.94
Low (≤ 15)	7.35	1.51

Table 1: Differential fertility and percentage share of private funding

A.2 Data on U.S. States

The National Center for Education Statistics provides direct general education expenditures per capita of U.S. state and local governments for the years 1997 and 1998. While we do not have estimates of private expenditures, we can investigate whether income inequality is a good predictor of public spending on education. Computing the correlation between the Gini inequality coefficient estimated in 1979 (U.S. Census Bureau) and the logarithm of public spending on education per capita in 1997-1998, we find that the correlation is negative and strong, -0.46. Figure 5 presents the corresponding cross plot with the Gini coefficient on the horizontal axis.

A.3 Public Education Spending and Democracy

Although private spending on education is available for only few countries, more data is available for the counterpart of private spending, namely public spending on education (measured as a fraction of GDP). Here we have a sample of 158 countries. We divide the country sample into 3 groups, based on their "level" of democracy. The democracy index is computed as a 5 years average (1989/90-1994/95) of the political right index from the Freedom in the World Country Ratings. This index lies on a one-to-seven scale, with one representing the highest degree of freedom. Public spending on education as a share of GDP is from the World Bank Development Indicators and are averaged over the period 1990-1995. Table 2 displays the mean and variance of public spending over GDP for the three groups of countries. Variance is decreasing with democracy, the mean follows a U shape. Table 3 displays two-tailed tests of whether the differences put forward by Table 2 are significant or not. The mean in democratic countries is significantly different from the one in the two other groups (medium and low), and the variance in the low democracy group is significantly higher than in the two other groups.

Democracy Index	Observations	Mean	Variance
High (= 1)	33	5.50	2.58
Medium ($1 < x \leq 4$)	57	4.23	3.01
Low ($4 < x \leq 7$)	68	4.56	4.95

Table 2: Public spending on education

	Mean Difference Test		Variance Ratio Test	
	Medium	Low	Medium	Low
High	1.27 [0.01]	0.94 [0.02]	0.86 [0.65]	0.52 [0.05]
Medium		-0.32 [0.36]		0.61 [0.06]

Table 3: Mean and variance tests

B Technical Appendix

Proof of Lemma 1: From budget constraint (2) total spending on children is given by $w^i \phi n^i + n^i e^i \bar{w}$. Substituting either $n^i = \hat{n}$ and $e^i = 0$ or $n^i = \tilde{n}$ and $e^i = \eta \phi x^i / (1 - \eta)$

yields that

$$w^i\phi n^i + n^i e^i \bar{w} = \frac{\gamma}{1+\gamma} w^i.$$

Taxable income therefore is

$$w^i(1-\phi n^i) - n^i e^i \bar{w} = \frac{1}{1+\gamma} w^i.$$

■

Proof of Lemma 2: We compute the level s_i such that group i is indifferent between public and private by solving $u^i[v, \hat{n}, 0, s_i] = u^i[v, \tilde{n}, e^i, 0]$:

$$s^i = (1-\eta)^{\frac{1}{\eta}-1} \eta\phi x^i.$$

s^i is bigger than zero and depends positively on x^i. If s is greater than s^i, $u^i[v, \hat{n}, 0, s_i]$ is greater than $u^i[v, \tilde{n}, e^i, 0]$, and type i prefer public education. Since $x^B > x^A$, we have $s^B > s^A$, which proves the Lemma. ■

Proof of Lemma 3: Using the results of Lemma 2, $\psi^B > 0$ implies $s \geq s^B > s^A$ which in turn implies $\psi^A = 1$. In the same way, $\psi^A < 1$ implies $s \leq s^A < s^B$, which in turn implies $\psi^B = 0$. ■

Proof of Proposition 1: Since $\Delta^A[\Psi]$ and $\Delta^B[\Psi]$ are continuous functions, $F(\Delta^A[\Psi], \Delta^B[\Psi])$ defines a continuous mapping from $[0,2]$ to $[0,2]$. By Brouwer's fixed point theorem, there is a fixed point $\Psi^\star$. The ψ^A and ψ^B corresponding to this $\Psi^\star$ satisfy (14) and hence an equilibrium exists. ■

Proof of Corollary 1: As Ψ goes to zero, we have that:

$$\lim_{\psi^A \to 0} s[\psi^A, 0] = \eta\phi.$$

With the quality of education going to $\eta\phi$, unskilled people prefer public education because $\Delta^A[0] = \gamma(-\eta \ln(x^A) - (1-\eta)\ln(1-\eta)) > 0$. Therefore the private regime $\Psi = 0$ cannot be sustained. ■

Proof of Proposition 2: We need to prove that $u^i[v, \hat{n}, 0, s] - u^i[v, \tilde{n}, e^i, 0]$ is monotonically decreasing in ψ^A and ψ^B. Since the terms with v cancel out, and the terms with n are constant, the only term that depends on ψ^A and ψ^B is s. It is thus sufficient to prove that s is monotonically decreasing in ψ^A and ψ^B. When varying ψ^A, we know by Lemma 3 that ψ^B is equal to 0. When varying ψ^B, we know from Lemma 3 that ψ^A is equal to 1. From equation (12) it appears immediately that

$$\frac{\partial s[\psi^A, 0]}{\partial \psi^A} < 0 \quad \text{and} \quad \frac{\partial s[1, \psi^B]}{\partial \psi^B} < 0.$$

■

Proof of Proposition 3: When $\Psi = 2$, the optimal schooling and taxation choice takes a simple form:

$$s = s[1,1] = \frac{\eta\phi}{1+\gamma\eta} \quad \text{and} \quad v = \frac{\gamma\eta}{1+\gamma\eta}.$$

In the segregation regime, all skilled people go to private schools and all unskilled ones go to public school: $\Psi = 1$. The public schooling level is given by $s = s[1,0]$, which yields equation (16). The quality of public schooling is higher in the segregation regime than in the public regime because $s[1,0] > s[1,1]$ for all $\xi^B \in]0,\infty[$. The quality of public schooling decreases (increases) in the relative number of unskilled (skilled) people in the population since $\partial s[1,0]/\partial\xi^B > 0$. ■

Proof of Proposition 4: For the public regime to be an equilibrium, a skilled person should weakly prefer public schooling to private schooling, given the tax rate and schooling level that prevails in this regime. If this condition is satisfied for a skilled parent, then by Lemma 3, the same follows for an unskilled parent. The constraint takes the form:

$$u^B[v,\hat{n},0,s] \geq u^B[v,\tilde{n},e^B,0].$$

After substituting v, $\hat{n}$, s, $\tilde{n}$ and e^B by their equilibrium values, the condition leads to (19). Thus, the relative income of skilled workers x^B should be below a certain threshold for them to stay in the public schools. Two cases can be distinguished. If this threshold is larger than one, the public regime will emerge if x^B lies between 1 and this threshold. If the threshold is lower than one (for example, if γ is large), then the public regime cannot arise in equilibrium.

For the segregated regime to arise in equilibrium, two participation constraints have to be met. First, the skilled persons should have no incentive to go to public schools, and second, the unskilled persons should have no incentive to go to private schools. The first constraint can be written as:

$$u^B[v,\hat{n},0,s] \leq u^B[v,\tilde{n},e^B,0].$$

After substituting v, $\hat{n}$, s, $\tilde{n}$ and e^B by their equilibrium values, the constraint leads to (17). Hence x^B has to be sufficiently large for the skilled persons to stay in the private schools. Intuitively, the gain from private schooling is a function of the income of the parent, with the gain the greater the greater the income. The right hand side of (17) tends to the one of equation (19) (public regime) as ξ^B goes to zero. It is increasing in ξ^B and concave, and tends to $\delta > 1$ as ξ^B goes to infinity. When $\gamma > \hat{\gamma}$, the constraint never binds for small ξ^B, but it can always be binding for large ξ^B since $\delta > 1$. More precisely, when $\gamma > \hat{\gamma}$, the right hand side of equation (17) starts below the line $x^B = 1$

for low ξ^B, crosses the line at the point:

$$\xi^B = \frac{\delta - (1+\gamma\eta)}{1-\delta}, \tag{30}$$

and then goes to δ.
The second constraint (unskilled parents do not choose private school) can be written as:

$$u^A[v, \hat{n}, 0, s] \geq u^A[v, \tilde{n}, e^A, 0].$$

After substituting v, $\hat{n}$, s, $\tilde{n}$ and e^A by their equilibrium values, and replacing x^A using equation (6), the constraint leads to (18). Here again the threshold $\hat{\gamma}$ plays a central role. If $\gamma > \hat{\gamma}$, the right hand side of equation (18) starts from $+\infty$ at $\xi^B = 0$, decreases, crosses the axis $x^B = 1$ at the point given in equation (30), and then converges to 1 from below as ξ^B goes to $+\infty$. Since the right hand sides of equations (17) and (18) cross the axis $x^B = 1$ at the same point, the two conditions cannot be violated at the same time. For $\gamma < \hat{\gamma}$, the constraints (18) never binds.

In the two partial segregation regimes, either the skilled or the unskilled parents are indifferent between public and private schooling. We first consider the case where the skilled are indifferent:

$$u^B[v, \hat{n}, 0, s] = u^B[v, \tilde{n}, e^B, 0].$$

The equilibrium values for ψ^B, s, and v are obtained by solving a system of three equations including the indifference condition, $s = s[1, \psi^B]$, and budget constraint (10). we obtain:

$$\begin{aligned} s &= \frac{\eta\phi x^B}{\delta}, \\ v &= 1 - \frac{x^B}{\delta}, \\ \psi^B &= \frac{(1+\xi^B)(\delta/x^B - 1) - \gamma\eta}{\gamma\eta\xi^B}. \end{aligned} \tag{31}$$

For partial segregation to occur in equilibrium, the only condition is that the value for ψ^B in (31) is between zero and one. It is also necessary that unskilled parents strictly prefer public schooling, but since the skilled are indifferent this condition is automatically satisfied because of Lemma 3. We therefore only need to check that ψ^B is between 0 and 1. Using equation (31), the constraint $0 < \psi^B < 1$ holds if:

$$\frac{\delta(1+\xi^B)}{1+\xi^B+\gamma\eta} > x^B > \frac{\delta}{1+\gamma\eta}.$$

This inequality is satisfied when the participation constraints (19) and (17) are simultaneously violated.

In the second type of partial segregation, it is the unskilled parents who are indifferent between the two types of schools:

$$u^A\left[v, \hat{n}, 0, s\right] = u^A[v, \tilde{n}, e^A, 0].$$

As above, the equilibrium values ψ^A, s and v are obtained by solving a system of three equations including the indifference condition, $s = s[\psi^A, 0]$, and budget constraint (10). We obtain:

$$\begin{aligned} s &= \frac{\eta\phi x^A}{\delta}, \\ v &= 1 - \frac{x^A}{\delta}, \\ \psi^A &= \frac{(1+\xi^B)(x^A - 1/\delta)}{\gamma\eta}. \end{aligned} \tag{32}$$

We only need to verify that ψ^A is below 1 (it is above 0 by Corollary 1), since by Lemma 3 the skilled always prefer private schooling when the unskilled are indifferent. Using equations (6) and (32), the constraint $\psi^A < 1$ holds if:

$$x^B < \frac{1+\xi^B}{\xi^B}\left(1 - \frac{\delta}{1+\xi^B+\gamma\eta}\right)$$

This inequality is satisfied when the participation constraint (18) is violated. ■

Proof of Proposition 5: It is sufficient to prove that $u^i[v, \hat{n}, 0, s] - u^i[v, \tilde{n}, e^i, 0]$ are monotonically decreasing in ψ^A and ψ^B for $\theta^A \geq 1$, which amounts as in Proposition 2 to proving that s is monotonically decreasing in ψ^A and ψ^B. When varying ψ^A, we know by Lemma 3 that ψ^B is equal to 0:

$$\frac{\partial s[\psi^A, 0]}{\partial \psi^A} = -\frac{\gamma\eta^2(\theta^A)^2(1+\xi^B)\phi}{(\theta^A + \xi^B + \theta^A\psi^A\gamma\eta)^2}$$

is always negative. When varying ψ^B, we know by Lemma 3 that ψ^A is equal to 1:

$$\frac{\partial s[1, \psi^B]}{\partial \psi^B} = \frac{-\eta\phi\xi^B(1+\xi^B)[\xi^B\psi^B\gamma\eta(\xi^B\psi^B + 2\theta^A) + \xi^B(\theta^A - 1) + \theta^A(\theta^A(1+\gamma\eta) - 1)]}{(1+\xi^B\psi^B)^2(\xi^B + \theta^A + \gamma\eta(\theta^A + \xi^B\psi^B))^2}.$$

The numerator is decreasing in ψ^B: if the numerator is negative for $\psi^B = 0$, then it is negative for all ψ^B. The numerator when $\psi^B = 0$ is equal to:

$$-\eta\phi\xi^B(1+\xi^B)[\xi^B(\theta^A - 1) + \theta^A(\theta^A(1+\gamma\eta) - 1)].$$

This expression is negative when condition (22) holds. In particular, it is negative for any $\xi^B \geq 0$ for $\theta^A \geq 1$. When θ^A is smaller than one, uniqueness of equilibrium depends on ξ^B. When the population weight of skilled people decreases, the condition on θ^A becomes less restrictive; at the limit, when $\xi^B \to 0$, the threshold tends to $1/(1+\gamma\eta)$. ■

Proof of Proposition 6: We start by observing that the level of public schooling in the public regime is independent of θ_A:

$$s_\theta[1,1] = s[1,1].$$

This implies that the public regime exists for all $\theta^A < 1$ if it also exists for $\theta^A = 1$. Hence, in the case $\gamma < \hat{\gamma}$, the public regime is an equilibrium for $x^B < \delta/(1+\gamma\eta)$ and for all $\theta^A \leq 1$. We have shown above that if θ^A is below the threshold stated in the proposition, private education is an equilibrium as well. Hence, there are at least two equilibria. Finally, to establish that there are at least three equilibria we can use the continuity of the functions Δ^A and Δ^B with respect to Ψ to prove the existence of a third equilibrium. For a given θ^A below the threshold, we have Δ^A and Δ^B smaller than 0 at $\Psi = 0$ and bigger than 0 at $\Psi = 2$. It follows from continuity and Lemma 3 that there is either a Ψ between 0 and one where $\Delta^A = 0$ and $\Delta^B < 0$, or a Ψ between 1 and 2 with $\Delta^A > 0$ and $\Delta^B = 0$. Thus there is a third equilibrium with partial segregation. ■

Proof of Proposition 7: At given ψ^A and ψ^B, the quality of public education increases with θ^A; taking the derivatives of s given in (20) yields:

$$\frac{\partial s_\theta[\psi^A,\psi^B]}{\partial\theta^A} = \frac{(\psi^A - \psi^B)\eta\phi\xi^B(1+\xi^B)}{(\psi^A + \psi^B\xi^B)\left[\theta^A + \xi^B + \eta\gamma(\psi^A\theta^A + \psi^B\theta^B)\right]^2} > 0.$$

It implies that the $\Delta^i[\Psi]$'s (utility difference between public and private education) are increasing with θ^A. Therefore, the mapping ${}^F(\Delta^A,\Delta^B)$ defined in the proof of Proposition 1 is weakly increasing in θ^A. Since equilibria are fixed points of F, the first part of the proposition follows. Turning to the equilibrium tax v given in (21), the derivatives are:

$$\frac{\partial v}{\partial\theta^A} = \frac{(\psi^A - \psi^B)\eta\phi\xi^B}{\left[\theta^A + \xi^B + \eta\gamma(\psi^A\theta^A + \psi^B\theta^B)\right]^2} \geq 0, \quad \frac{\partial v}{\partial\psi^A}|_{\psi^B=0} = \frac{\gamma\eta\theta^A(\theta^A + \xi^B)}{(\theta^A + \xi^B + \gamma\eta\theta^A\psi^A)^2} > 0,$$

$$\frac{\partial v}{\partial \psi^B}\Big|_{\psi^A=1} = \frac{\gamma\eta\xi^B(\theta^A + \xi^B)}{\left[\theta^A + \xi^B + \gamma\eta(\theta^A + \psi^B\xi^B)\right]^2} > 0.$$

Hence the total effect of increasing θ on v is non-negative. In fact, it is strictly positive except at $\Psi = 2$. ■

Proof of Proposition 8: Since there is at each date t an equilibrium of period t that does not depend on the future, and since the dynamics (24) are defined in $\mathbb{R}^2_+$, the existence and uniqueness of time-t equilibria implies the same properties for intertemporal equilibria. ■

Proof of Proposition 9: We compute the limits of the function Γ defined in equation (26) when ξ^B_t goes to zero and to infinity; both $\Gamma(0)$ and $\lim_{\xi\to\infty}\Gamma(\xi)$ are strictly positive and finite. Since Γ is a continuous function on $\mathbb{R}_+$ and converges to finite values on the border of its definition set, it is bounded from above. Hence, dynamics of ξ^B_t are bounded. Since $\Gamma(0) > 0$ and $\Gamma(\xi) < \xi$ for large ξ, there is a least one $\bar{\xi}$ such that $\Gamma(\bar{\xi}) = \bar{\xi}$ and $\bar{\xi}$ is a steady state. ■

Proof of Proposition 10: The upper bound on w^B is defined by the participation constraint (19). Linearizing (27) around the steady state, we find that the condition

$$|(\tau^B - \tau^A)s[1,1]^\eta| < 1$$

is a necessary and sufficient condition for local stability. Since $\tau^B s[1,1]^\eta < 1$ and $\tau^A s[1,1]^\eta < 1$, this condition always holds. ■

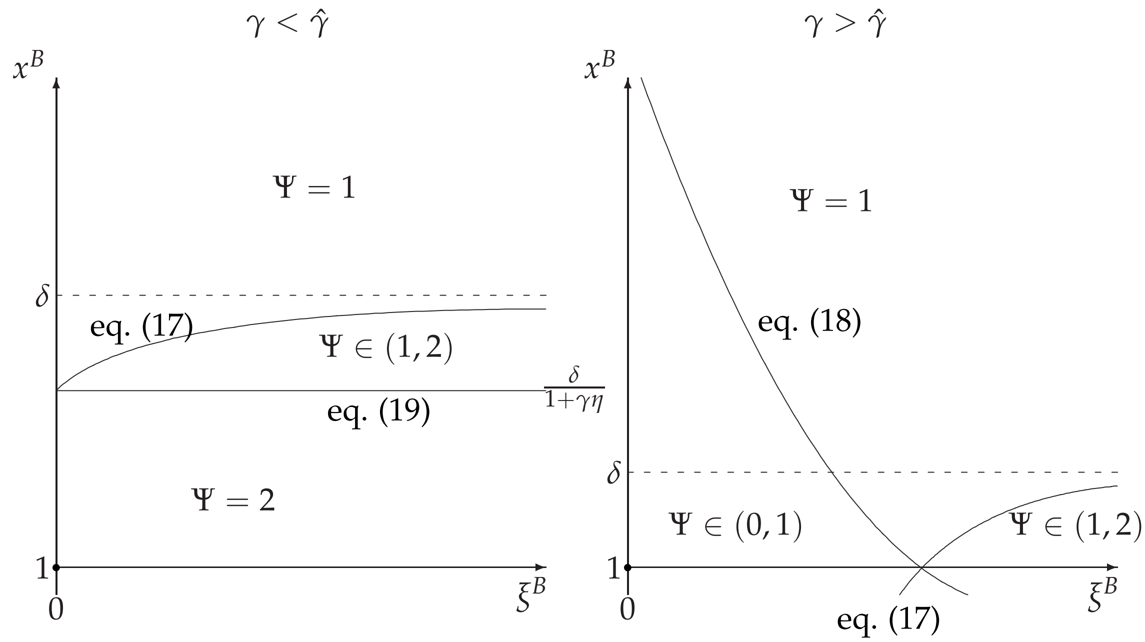

Figure 1: The education regimes

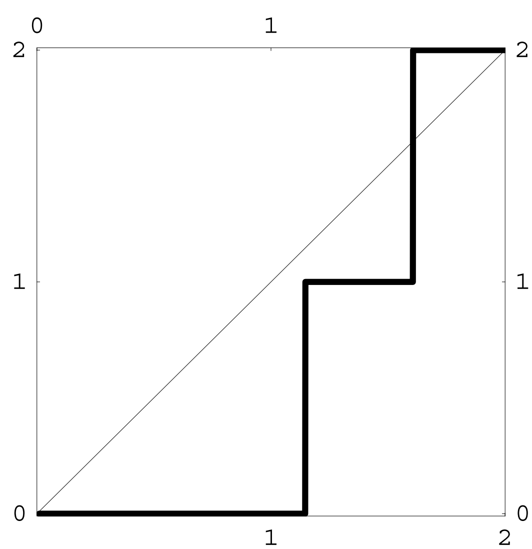

Figure 2: The mapping F for $\eta = 0.2$, $\phi = .075$, $\gamma = 2$, $w^B = 3$, $\xi^B = 0.6$, and $\theta^A = 0.01$.

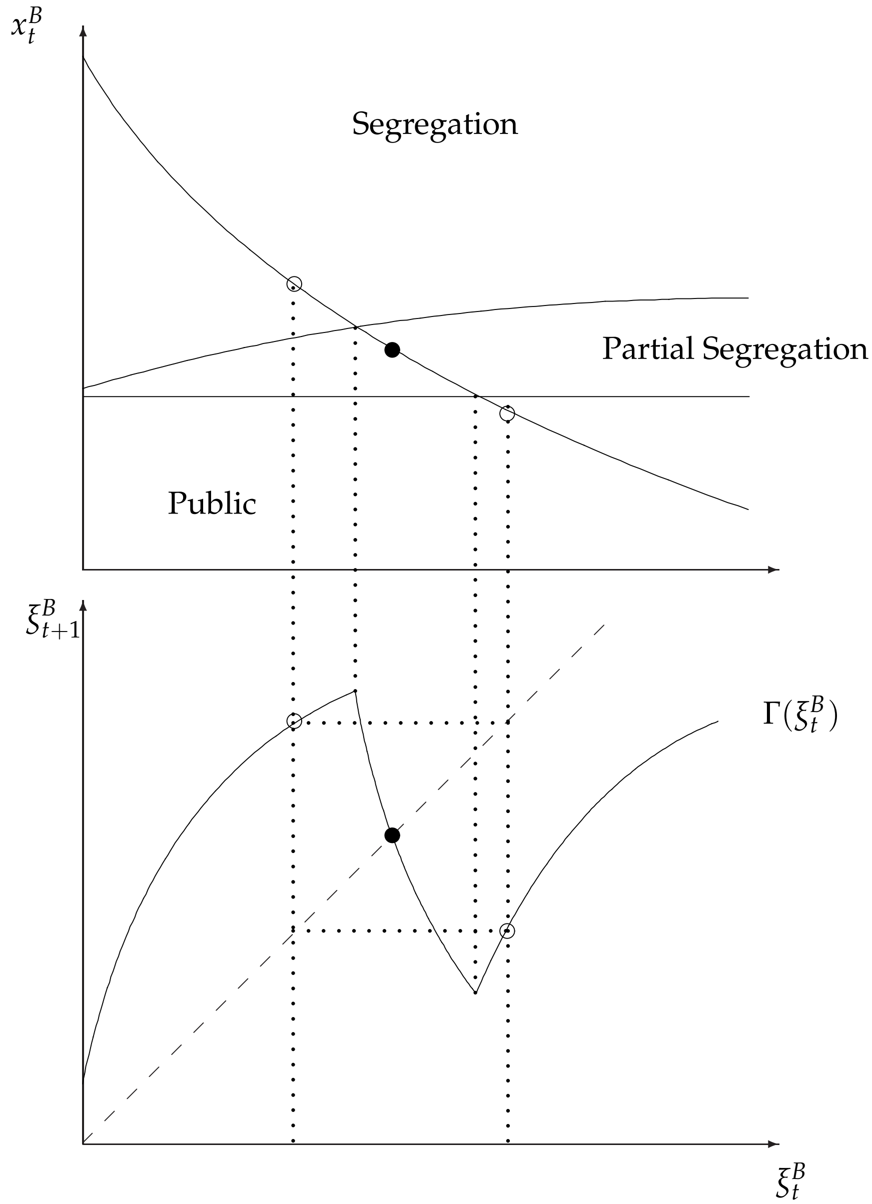

Figure 3: Example of a period-2 cycle

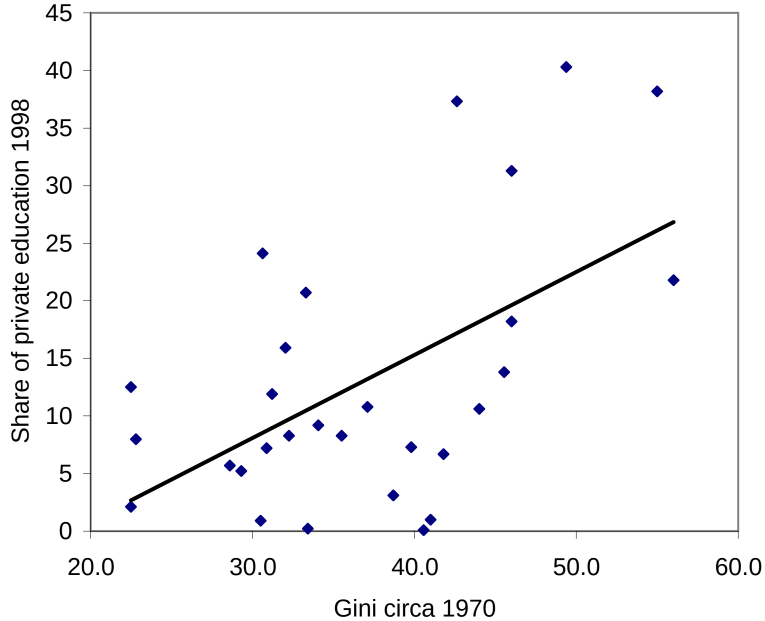

Figure 4: Inequality and education systems across countries

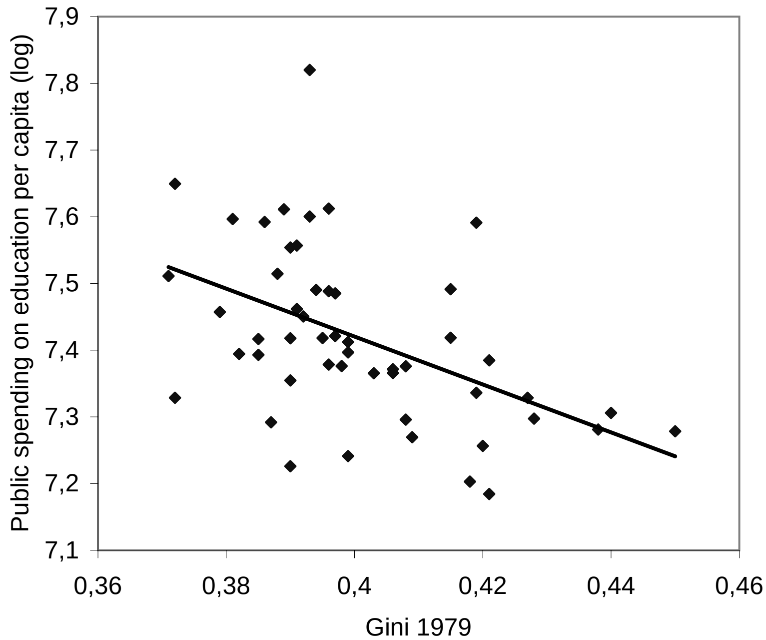

Figure 5: Inequality and education across U.S. states